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On

“Speculation of some anisotropic cosmological models in modified theories of gravitation”

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Speculation of some anisotropic cosmological models in modified theories of gravitation

1. Introduction:

Cosmology is the study of the whole universe. The universe is very vast and collection of galaxies. Cosmology deals with this large scale structure. The enigma of the universe is attracted human being from long ago. Is the universe finite or infinite?. What is beginning and end of the universe?. These are some of the crucial questions trying to solve cosmologist from thousands of years. Newton's law of gravitation is milestone in the history of cosmology. In 1915, Einstein developed general theory of relativity. According to the general theory of relativity, gravitation is nothing but wrapping of space time. It is very successful theory. Many cosmological phenomenons are explained with the help of general relativity.

General Relativity is based on the principle of equivalence, the principle of covariance and Mach's principle. The general field equations i.e., Einstein's field equations which are associated with gravitational field are represented by

$$R_i - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -8\pi G T_{ij},$$

where R_{ij} is the Ricci tensor, R is Ricci scalar, T_{ij} is the energy momentum tensor, Λ is the cosmological constant and G is the Newton's constant of gravitation.

However, Einstein himself modified it by introducing cosmological constant in the field equations of general relativity. General theory of relativity fails to explain problems like initial singularity, horizon and flatness. General

theory of relativity is modified by introducing the term *f(R)* in Einstein- Hilbert action. It is known as *f(R)* theory of gravitation. *f(G), f(R,G),f(T), f(Q)* are some of the modified theories of gravitations. *f(R)* models are equivalent to Brans-Dicke theory with coupling parameter $\omega = 0$. Cosmological models in *f(R)* theory do not have normal matter- dominated epoch. Recently, Harko et al. (2011) have proposed more general model of modified gravity theory *f(R,T)* where *R* is Ricci Scalar and trace *T* of the energy momentum tensor.

The present universe is isotropic at large scale. Notwithstanding, cosmological observational data such as cosmic background explorer and Wilkinson microwave anisotropy probe supports the existence of an anisotropic phase of the evolution of the universe. The early anisotropy of the universe is decay into present isotropy. These studies lead many cosmologists to consider anisotropic cosmological models. Bianchi type I-IX models are play an important role in the investigations of anisotropic universe. These models are homogeneous and anisotropic.

2. Review of Literature

In the last decades the several theories of gravitation have been proposed as an alternative to Einstein's general theory of relativity. Schmidt (2008) studied the effect of modification to general relativity on large-scale weak lensing observables in that they considered three modified gravity scenarios: $f(R)$ gravity, the Dvali-Gabadadze-porrati model and tensor-vector-scalar theory. Ballardini and Finelli (2022) investigated the use of type of Ia Supernovae (SNeIa) in the context of scalar-tensor theories of gravity, with working example induced gravity, equivalentto Jordan-Dicke theory. Martino, er al. (2014) studied $f(R)$ theory of gravitation and compute pressure profile of 579 galaxy clusters assuming that the gas is in hydrostatic equilibrium within

the potential well of the modified gravitational field and they found that the resulting profiles of these systems fit the data without requiring a dominant dark-matter component with model parameters similar to those required to explain the dynamics of galaxies. Calabrese et al. (2008) analysed WMAP 3-years and ACBAR cosmic microwave anisotropies angular power spectra and it shows evidence for gravitational lensing at >3σ level. Amendola et al. (2014), studied how modified gravity (MG) affects B-mode spectra and identify two main observable effects: lensing and propagation of tensor mode. Rigorous distinction between dark energy and modified gravity based on the strong and weak equivalence principles is introduced by Joyce et al (2016). Koivisto et al. (2009) investigated whether inflation, either isotropic or anisotropic, may be supported by n-forms.they found new isotropic solution for three forms and Anisotropic solution for two forms, generalising vector inflation.

Sahoo et al. (2017) studied spatially homogeneous anisotropic Bianchi type-I universe in $f(R,T)$ gravity (where R is Ricci Scalar and T is the trace of stress energy momentum tensor) in two different cases viz. $f(R,T) = R + 2f(T)$ and $f(R,T) = f_1(R) + f_2(T)$ with bulk viscosity matter content for this they considered a time varying deceleration parameter (DP), which generates an accelerating universe to obtain the exact solution of the field equations. Shamir (2016) studied the anisotropic universe in $f(G)$ Gravity for that locally rotationally symmetric Bianchi type I spacetime is considered and A viable $f(G)$ model is used to explore the exact solutions of modified field equations. In particular two families involving power law and exponential type solutions have been discussed .by Valentino et al.(2012), adopted a parametrized approach and focused on three different classes of model: Linder's growth index, Chameleon-type model, and *f(R)* theories.

Energy conditions in $f(R,G)$ gravity studied by Atazadeh and Darabi (2014). They constructed the inequalities obtained by energy conditions and specially apply the weak energy condition using the recent estimated values

of the Hubble, deceleration, jerk and snap parameters to probe the viability of two forms of$f(R,G)$. Sahoo et al. (2020) analysed $f(R,G)$ gravity model with a single parameter studied the strong energy condition (SEC), the weak energy condition (WEC), the null energy condition (NEC) and the dominant energy condition (DEC) under the simplest non-minimal matter geometry coupling with a perfect fluid distribution.

Tilted cosmological models in $f(R,T)$ theory of gravitation are investigated by Pawar and Dagwal (2015). In the context of $f(R,T)$ theories of gravity, the evolution of scalar cosmological perturbations in the metric formalism studied by Alvarenga et al. (2013). They found complete set of differential equations for the matter density perturbations. Pawar et al. (2014), investigated tilted plane Symmetric cosmological models in presence and absence of magnetic field . Katore and Hatkar (2015), considered the Friedmann-Robertson-Walker universe in the frame work of $f(R)$ gravity and investigated interacting & non-interacting cases of the two fluids. They found that the universe is closed in the non-interacting Zeldovich fluid model and it is open at the present in the dust fluid model, whereas in interacting models, there is energy transfer from the radiation era to the matter era. Board and Barrow (2017) studied the cosmological effects of adding terms of higher order in the usual energy-momentum tensor to the matter Lagrangian of general relativity and they found a range of exact solutions for isotropic universes, discussed their behaviours with reference to the early- and late-time evolution, accelerated expansion, and the occurrence or avoidance of singularities. Pradhan et al. (2006), investigated FRW models of the universe based on Lyra's manifold. A new class of exact solutions has been obtained by considering a time dependent displacement field for variable deceleration parameter from which three models of the universe are derived (i) exponential (ii) polynomial and (iii) sinusoidal form respectively. Rahaman et al. (2015), obtained a Cosmological model with a viscous fluid in Kaluza-Klein metric by assuming a time-dependent equation

of state. Koivisto and Mota (2008) investigate cosmologies where the accelerated expansion of the Universe is driven by a field with an anisotropic equation of state. Katore et al. (2014), explored hypersurface homogeneous space time in Brans Dicke theory of gravitation in terms of the dark energy source and obtained solution of the field equation.

3. Aim:

Our aim is to address some anisotropic cosmological models with modified gravity theories foreground.

4. Objective:

1. To extend the present work.
2. Analysis some anisotropic cosmological models in the context of modified theories of gravitation
3. To Study the higher dimensional cosmological models in the context of modified theory of gravitation
4. To investigate the physical, geometrical, kinematical properties and applications of existing models.
5. To investigate cosmological models in the presence of various physical fluids like perfect fluid, viscous fluid, dark energy, string, domain walls etc.
6. To ascertain the reliability, consistency and compatibility of the theories under investigation as regard to their predictions in view of observational data.

5. Methodology

For the proposed entitled work, we will collect the relevant documents such as research papers, reference books from various resources like reputed e-libraries, NISCAIR, New Delhi, University Library etc. We will analyse the research papers which are published in reputed journals relevant to

the research proposed. Further, we will do Mathematical computational work for solving the field equations employing the existing methods of solving partial differential equations. In case of difficulties the numerical method may be adopted. We will construct and present the research papers in workshops/conferences/seminars for up gradation of quality of our research papers.

6. Important of Study

From ancient times the universe has become matter of curiosity and wonder to the human kind. There are so many interesting things in the universe. The main purpose of this work is to examine and explore the hidden aspects of the universe. Our investigation may lead to new cosmological models which will help to decide the importance of gravitational theories.

7. Plan of Research Work:

Sr.No	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic.	Good command on the subject and clear the concepts.
2	Second year	Collecting the papers and analyze them.	To understand the mathematical techniques of Research Paper
3	Third Year	Work on some problems to be included in the thesis.	Submit at least two papers.

4	Fourth Year	Publishing some papers and submitting the Thesis.	Some more papers and Thesis submission.

8 . References:

1. T. Clifton , J. D. Barrow,Physical Review D 72, 103005 (2005)
2. T. Koivisto , D. F. Mota, arXiv:.v1 [Astro-ph], (2008)
3. M. F. Shamir, Astrophys Space sci, 316:147 (2016)
4. P. K. Sahoo,Parbati Sahoo , B. K. Bishi, World Scientific,Vol. 14, 1750097 (2017)
5. De Martino, M. De Laurentis, F. Atrio-Barandela , S. Capozziello, MNRAS 442, 921-928 (2014)
6. E. Calabrese, A. Sloar, A. Melchiorri, G. F. Smoot , O. Zahn, Physical Review D 77, 123531 (2008)
7. F. Rahaman, B. C. Bhui& B. Bhui, Astrophysics Space Science 301:47-49 (2006)
8. C. V. R. Board & J. D. Barrow, Physical Review D 96 123517 (2017)
9. A. Pradhan, J. P. Shahi & C. B. Singh, B. J. of Physics, Vol. 36, no. 4A, (2006)
10. A. Joyce, L. Lombriser& F. Schmidt, Annu. Rev. Nucl. Part. Sci. 66:95-122 (2016).
11. F. G. Alvarenga, A. sDe la C ruz-Dombriz, M. J. S. houndjo, M. E. Rodrigues , D. Saezz-Gomez, Physical Review D 87, 103526 (2013)
12. L. Amendola, G. Ballesteros 7 V. Pettorino,Physical Review D 90, 043009 (2014)
13. K. Atazadeh , F. Darabi, Gen RelativGravit 46:1664 (2014)
14. P. K. Sahoo, S. Mandal , S. Arora, Astron. Nachr. / AN., 342:89–95 (2021)
15. S. D. Katore, M. M. Sancheti, S. P. Hatkar , N. K. Sarkate, Commun. Theor. Phys. 62, 768–74 (2014)
16. E. Di Valentino, A. Melchiorri, V. Salvatelli , A. Silvestri, Physical Review D 86, 063517 (2012)
17. D D Pawar ,V. J. Dagwal, prespacetime Journal Vol.07 Issue-02, (2015)
18. D. D. Pawar, V. J. Dagwal& Y. S. Solanke, prespacetime Journal, Vol-05 Issue-05 (2014)
19. S. D. Katore , S. P. Hatkar, Indian J Phys, (2015)
20. M. Ballardini , F. Finelli, arxiv:2112.15126v2 [Astro-ph.CO] (2022)
21. F. Schmidt, Physical Review D 78, 043002 (2008)

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